MATH 201: Calculus and Analytic Geometry III

Fall 2018-2019, Exam 2, Duration: 60 min.

Problem	1	2	3	4	5	6	Total
Points	14	21	15	10	18	22	100
Scores							

INSTRUCTIONS:

(a) Explain your answers precisely and clearly to ensure full credit.
(b) Closed book. No notes. No calculators. No cellphones.
(c) UNLESS CLEARLY SPECIFIED OTHERWISE, THE BACKSIDE OF THE PAGES WILL NOT BE GRADED.

Problem 2

(a) (7 pts) Does

$$\lim_{(x,y)\to(0,0)} \frac{2y^2 \sin^2 x}{\sqrt{x^4+y^2}}$$

exist? Justify your answer.

(b) (7 pts) What about

$$\lim_{(x,y)\to(0,0)} \frac{xy}{y-2x} \; ?$$

Justify your answer.

(c) (7 pts) Let

$$f(x,y)=\begin{cases} \dfrac{x^4y^4}{x^2+y^2} & \text{if } (x,y)\neq(0,0) \\ 0 & \text{if } (x,y)=(0,0) \end{cases}$$

Decide if $f(x,y)$ is continuous at (0, 0) or not. Justify your answer.

Problem 4

(10 pts) Let $f(x,y,z)$ be a differentiable function of three variables. Suppose that

$\nabla f(2,1,2) = 6\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ and $\nabla f(1,1,2) = \mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$.

Let $\begin{cases} x = r^2 + s \\ y = \dfrac{r}{s} \\ z = 2r + \ln\dfrac{r}{s} \\ \text{and } w(r,s) = f(x,y,z) \end{cases}$

Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial s}$ at the point $(r,s) = (1,1)$.

Problem 6

Let L be the line $y = 2x - 7$ in the xy-plane.

Suppose $f(x, y)$ is a differentiable function of two variables satisfying

- $f(x, y) = 2$ for all $(x,y) \in L$.
- $\nabla f(0,-7) = 4\mathbf{i} - 2\mathbf{j}$.
- $D_u f(3,-1) = \sqrt{5}$, where $u = \frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$.

(a) (10 pts) Estimate $f(0.03, -6.96)$.

(b) (8 pts) Find $\nabla f(3,-1)$.

(c) (4 pts) Suppose that f is **also constant** on the line $y = x - 2$. Find $\nabla f(5,3)$.